Valued Constraint Satisfaction Problem and Resilience in Database Theory

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joint work with Manuel Bodirsky and Carsten Lutz

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LICS
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Resilience of queries

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Conjunctive query: a formula q of the form $\exists y_1, \dots, y_l \, (\psi_1 \wedge \dots \wedge \psi_m)$, where ψ_i are atomic

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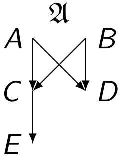
$$q = \exists x, y, z(R(x, y) \wedge R(y, z))$$

with respect to $\mathfrak{A}$ is 1 – remove (C, E).



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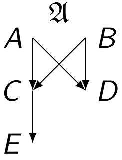
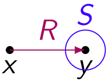
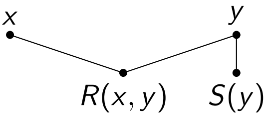


Goal: Classify complexity of resilience for all q.

Translation to a dual problem

Example:
$q := \exists x, y(R(x, y) \wedge S(y))$





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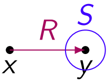

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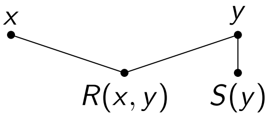

incidence graph I(q)

Theorem (Cherlin, Shelah, Shi '99)

Let q be a query and $\mathfrak{Q}$ its canonical structure. If $I(q)$ is connected, then there exists a structure $\mathfrak{B}_q$, such that for every finite $\mathfrak{A}$:

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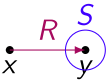

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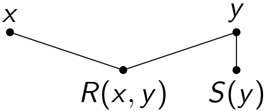

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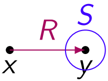

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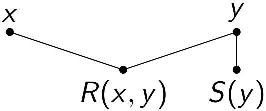

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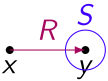

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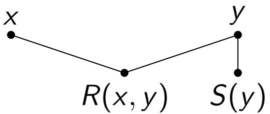

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oligomorphic – countable domain B_q and the action of $\mathrm{Aut}(\mathfrak{B}_q)$ on B_q^n has finitely many orbits for every $n \geq 1$

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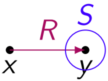

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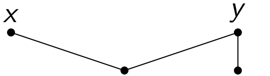

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Example: For every finite directed graph G we have:

 $\not\to G \Leftrightarrow G \to$ 

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A valued structure Γ consists of:

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Definition (VCSP(Γ))

Input: $u \in \mathbb{Q}$, an expression

$$\phi(x_1, \ldots, x_n) = \sum_i \psi_i,$$

where each ψ_i is an atomic τ-formula
Output: Is

$$\inf_{\bar{a} \in D^n} \phi(\bar{a}) \leq u \text{ in } \Gamma?$$

Connection of resilience and VCSPs

query q with $I(q)$ connected (WLOG) $\rightsquigarrow$ obtain the dual structure $\mathfrak{B}_q$ $\rightsquigarrow$ turn it into a valued structure Γ_q with cost functions taking values 0 and 1

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Corollary (Bodirsky, Lutz, S.)

Let q be a conjunctive query such that $I(q)$ is acyclic. Then the resilience problem for q is in P or NP-complete.

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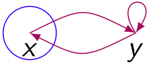

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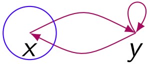

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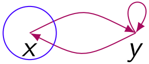

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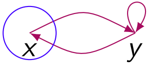

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- equivalently, for every query q such that $I(q)$ is a tree

Thank you for your attention

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